

Задание 1

Пусть $A(x)$ и $B(x)$ – переменные предикаты, а C – переменное высказывание (или формула, не содержащая x).

1. $\overline{\forall x A(x)} \equiv \exists x \overline{A(x)}$.
2. $\overline{\exists x A(x)} \equiv \forall x \overline{A(x)}$.
3. $\forall x \overline{A(x)} \equiv \overline{\exists x A(x)}$.
4. $\exists x \overline{A(x)} \equiv \overline{\forall x A(x)}$.
5. $\forall x A(x) \wedge \forall x B(x) \equiv \forall x [A(x) \wedge B(x)]$
6. $C \wedge \forall x B(x) \equiv \forall x [C \wedge B(x)]$.
7. $C \vee \forall x B(x) \equiv \forall x [C \vee B(x)]$
8. $C \rightarrow \forall x B(x) \equiv \forall x [C \rightarrow B(x)]$
9. $\forall x [B(x) \rightarrow C] \equiv \exists x B(x) \rightarrow C$.
10. $\exists x [A(x) \vee B(x)] \equiv \exists x A(x) \vee \exists x B(x)$.
11. $\exists x [C \vee B(x)] \equiv C \vee \exists x B(x)$.
12. $\exists x [C \wedge B(x)] \equiv C \wedge \exists x B(x)$.

Задание 2

1. $\forall x (F(x) \& G(x)) \equiv \forall x F(x) \& \forall x G(x)$
2. $\exists x (F(x) \vee G(x)) \equiv \exists x F(x) \vee \exists x G(x)$
3. $\forall x \forall y F(x, y) \equiv \forall y \forall x F(x, y)$
4. $\exists x \exists y F(x, y) \equiv \exists y \exists x F(x, y)$
5. $\neg(\forall x F(x)) \equiv \exists x \neg F(x)$
6. $\neg(\exists x F(x)) \equiv \forall x \neg F(x)$

Задание 3

$((\exists z A(z) \rightarrow (\exists x J(x) \sim \forall y Q(y))) \& (((\exists x M(x) \rightarrow \exists x P(x)) \sim \forall y Q(y)))$

$\equiv ((\neg(\exists z A(z)) \vee (\exists x J(x) \sim \forall y Q(y))) \& ((\neg(\exists x M(x)) \vee \exists x P(x)) \sim \forall y Q(y)))$

$\equiv ((\neg(\exists z A(z)) \vee \neg(\exists x J(x) \vee \forall y Q(y))) \& (\exists x (x) \vee \neg(\forall y Q(y)))) \& ((\neg(\exists x M(x)) \vee \exists x P(x)) \vee \forall y Q(y)) \& ((\neg(\exists x M(x)) \vee \exists x P(x)) \vee \neg(\forall y Q(y)))$

$\equiv ((\forall z \neg A(z) \vee (\forall x \neg J(x) \vee \forall y Q(y))) \& (\exists x (x) \vee \exists y \neg Q(y))) \& (\exists x M(x) \vee \exists x P(x) \vee \forall y Q(y)) \& (\forall x \neg M(x) \vee \exists x P(x) \vee \exists y \neg Q(y))$

Задание 4

$$\begin{aligned} & ((\exists x \forall y P(x,y)) \rightarrow (\forall x \forall y W(x,y))) \& (\exists x \forall y P(x,y) \sim \forall x \exists y R(x,y) \rightarrow \exists x \exists y \\ & M(x,y) \sim \forall x \forall y W(x,y)) \\ & \equiv (\neg(\exists x \forall y P(x,y)) \vee (\forall x \forall y W(x,y))) \& (\exists x \forall y P(x,y) \sim \neg(\forall x \exists y R(x,y)) \vee \exists x \exists y \\ & M(x,y) \sim \forall x \forall y W(x,y)) \\ & \equiv (\neg(\exists x \forall y P(x,y)) \vee (\forall x \forall y W(x,y))) \& ((\neg(\exists x \forall y P(x,y)) \vee \neg(\forall x \exists y R(x,y))) \& \\ & (\exists x \forall y P(x,y) \vee \neg(\forall x \exists y R(x,y))) \vee (\neg(\exists x \exists y M(x,y)) \vee (\forall x \forall y W(x,y))) \& ((\exists x \exists y \\ & M(x,y) \vee \neg(\forall x \forall y W(x,y)))) \\ & \equiv (\forall x \exists y \neg P(x,y) \vee \forall x \forall y W(x,y)) \& ((\forall x \exists y \neg P(x,y) \vee \exists x \forall y \neg R(x,y)) \& (\exists x \forall y \\ & P(x,y) \vee \forall x \exists y R(x,y)) \vee \forall x \forall y \neg M(x,y) \vee \forall x \forall y W(x,y)) \& (\exists x \exists y M(x,y) \vee \exists x \exists y \\ & \neg W(x,y))) \end{aligned}$$

Задание 5

Определение непрерывности функции в точке.

Функция $f(x)$, определенная на множестве E , непрерывна в точке $x_0 \in E$, если $\forall \varepsilon > 0 \exists \delta > 0 \forall x \in E (P(\varepsilon, \delta, x))$, где $P(\varepsilon, \delta, x) = (0 < |x - x_0| < \delta \rightarrow |f(x) - f(x_0)| < \varepsilon)$.