

```

.nr HM 0.3i
.nr FM 0.3i
.TL
.B "Mathematical analysis"
.EQ
gsiize 11
define prt 'partial'
delim $$
.EN
.SH
Scalar field
.PP
If each point of the field is a scalar value  $U$  or  $U=U(x,y,z)$  that the field is the scalar field.
.SH
Gradient
.PP
Gradient of scalar field:
.PP

$$\text{grad } U = \frac{\partial U}{\partial x} \mathbf{i} + \frac{\partial U}{\partial y} \mathbf{j} + \frac{\partial U}{\partial z} \mathbf{k}$$

.SH
Divergence
.PP
Divergence of vector field is:
.PP

$$\text{div } \mathbf{a} = \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}$$

.SH
Rotor
.PP
Rotor of vector field:
.PP

$$\text{rot } \mathbf{a} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ a_x & a_y & a_z \end{vmatrix}$$

.SH
Theorem of Gauss
.PP
Gauss's theorem (Ostrogradsky's theorem, divergence theorem) states that the outward flux of the vector field through a closed surface is equal to the volume integral of the divergence over the region inside the surface.

$$\int_V \text{div } \mathbf{a} \, dV = \int_S \mathbf{a} \cdot \mathbf{n} \, dS$$

.SH
Solenoidal vector field
.PP
If each point of the vector field  $\mathbf{a}$  corresponds to the condition  $\text{div } \mathbf{a} = 0$ , it means this field is solenoidal.
.SH
Potential vector field
.PP
If each point of the vector field  $\mathbf{a}$  corresponds to the condition  $\text{rot } \mathbf{a} = 0$ , it means this field is potential.
.SH
Rotational vector field
.PP
If  $\text{rot } \mathbf{a} \neq 0$  then the vector field  $\mathbf{a}$  is rotational.
.TS
tab(@), doublebox;
cw(1i) sw(1i) sw(2i)
c | l | l
^ | l | l
^ | l | l
c | s | l.
Properties@@
=
Vector field@Solenoidal@T{

$$\text{div } \mathbf{a} = 0$$


$$\text{grad } \mathbf{a} \cdot (\text{grad } \mathbf{a} \times \mathbf{a}) = 0$$

T}
@Potential@T{

$$\text{rot } \mathbf{a} = 0$$

T}
@Rotational@T{

$$\text{rot } \mathbf{a} = \text{grad } \mathbf{a} \times \mathbf{a} \neq 0$$

T}
=
Scalar Field@T{

$$U = U(x,y,z)$$

T}
.TE

```